Computation Strategies for Basic Number Facts

+, -, x, ÷

Addition
Subtraction
Multiplication
Division

Proficiency with basic facts aids estimation and computation of multi-digit numbers. The enclosed strategies provide ways of thinking about each operation (+, -, x, ÷) that facilitate the development of accurate, efficient, and flexible computation.

Basic Facts

Addition Strategies

There are broad categories of addition strategies - Count Up, Doubles, and Tens - that promote important understandings of number relationships and help children master basic facts. Students develop fluency as they think through these approaches, express strategies in their own ways, and ultimately discover the methods that work best for them.

<u>Count Up:</u> To add a small quantity on to a larger quantity.
For 14 + 3, **count on** fourteen and say, "*15, 16, 17*".

N + 0: Plus zero always equals the number
N + 1: Plus one is always one more than the number
N + 2: Count up two from the number
N + 3: Count up three from the number
[N means number.]

Encourage counting up when adding small numbers such as 1, 2, or 3.
[Although *counting up* can be used to join larger numbers together, it is time consuming. What follows are approaches to computation with larger addends that promote efficiency, accuracy, and speed.]

<u>Doubles:</u> **Doubles Facts** are relatively easy to learn because all the sums are even and make a counting by twos pattern. Doubles serve as "anchor facts" for related facts.

1 + 1 = 2	6 + 6 = 12
2 + 2 = 4	7 + 7 = 14
3 + 3 = 6	8 + 8 = 16
4 + 4 = 8	9 + 9 = 18
5 + 5 = 10	10 + 10 = 20

Doubles Plus One is also called Double Neighbors because one adds the number to the number that "lives next door." In other words, double the addend and add one more. This strategy can be extended to **Doubles Plus Two**.

Doubles Fact		Doubles Plus One
If you know 6 + 6 = 12	→	then 6 + 7 is just 6 + 6 + 1 = 13.
If you know 8 + 8 = 16	→	then 8 + 9 is just 8 + 8 + 1 = 17.

<u>Tens:</u> **Ten's Partners** are number pairs that equal 10. Seeing numbers in relationship to *making ten* is important for efficient computation in a base-10 number system.

0 + 10 = 10	3 + 7 = 10
1 + 9 = 10	4 + 6 = 10
2 + 8 = 10	5 + 5 = 10

Plus Ten comes easily for children once they have enough experience to understand and express the following pattern: When adding 10 to a number, the digit in the tens place increases by 1 while the digit in the ones place remains the same (however, when adding 10 to a number in the 90's the pattern changes resulting in 10 tens or 100).

Plus Nine is based on the fact that 9 is just one away from 10.
Two common approaches are:

See Nine, Think Ten: See 6 + 9, think 6 + 10 - 1.
See 14 + 9, think 14 + 10 - 1.

Turn Nine Into Ten: Turn 6 + 9 into 5 + 10.
Turn 14 + 9 into 13 + 10.

The above strategies can be applied to **Plus Eight** facts.

<u>Hidden Facts:</u> For other addition facts, we encourage students to find **Hidden Doubles** or **Hidden Tens** within these problems, thereby building on what they know. See how the addends are decomposed below to form doubles and tens that make computation easier.

Find the Hidden Helpers

	HIDDEN DOUBLES	HIDDEN TENS
7 + 5 = ?	**5 + 5** + 2 **6 + 6**	**7 + 3** + 2
6 + 8 = ?	**6 + 6** + 2 **7 + 7**	**6 + 4** + 4 **8 + 2** + 4

Key Terms

The discussion of basic number fact strategies will, by nature, highlight the properties of each operation and the learning of specific vocabulary which includes:

Addend:
The numbers in an addition problem.

3 + **4** = **7** ← sum
↑ ↑
addends

Commutative Property of Addition:
Numbers can be added on any order without changing the sum.

Digit:
The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used to write any number.

Fact Family:
A group of related addition and subtraction facts.

FAMILY OF FACTS
6 = 5 + 1 **6 – 5 = 1**
6 = 1 + 5 **6 – 1 = 5**

Identity Property of Addition:
When you add zero to a number, it does not change the total.

Place Value:
The value of a digit as determined by its position (the ones column, tens column, and so forth).

Sum:
The result of joining quantities; the total amount.

Basic Facts

Subtraction Strategies

As students examine subtraction and explore number relationships, they become able to make important generalizations about how subtraction works. Summarized below are several ways of thinking about subtraction that promote computational fluency and fact mastery.

Key Terms Subtraction is the comparison of quantities.	**Minuend** ↓ (16) **Difference** ↓ (9) $16 - 7 = 9$ ↑ (7) **Subtrahend**

<u>The difference can be calculated by counting back or counting up</u>.

Count Back: **Also thought of as *take away*, this approach is best when the subtrahend is small (-1, -2, -3).**
For 19-2, start at 19 and count back two to get to 17.

Count Up: **Count up from the lower number to *find the difference* between the two quantities**. **This method is best when the minuend and subtrahend are close together.**
For 99-97, count up two from 97 to 99.

<u>Subtraction can be thought of as addition</u>.

Think Addition: **Turn subtraction into addition problems.**
Turn 17 – 12 = ☐ into 12 + ☐ = 17
which is said as: *12 plus what equals 17?*

Ten's Partners: **When you know the sums of ten, you know the related subtraction facts:** 10 – 9 = 1, 10 – 8 = 2, 10 – 7 = 3, etc.

Double Facts: **By knowing your double facts, you also know the related subtraction facts:** 18 – 9 = 9, 16 – 8 = 8, 14 – 7 = 7, etc.

Ten/Nine/Eight: **Just as students learn patterns with +10, they apply the opposite patterns to -10. This can be extended to -9 and -8:**

See 9, Think 10: For -9, subtract 10 and add 1.
See 8, Think 10: For -8, subtract 10 and add 2.

<u>Subtract in small steps (to subtract through 10 or a multiple of 10):</u>

Split Subtrahend in Small Parts for Easier Computation: For 15 – 6, try 15 – 5 – 1.
For 24 – 7, try 24 – 4 – 3.

Basic Facts

Multiplication Strategies

Students' understanding of multiplication and division develops over time, generally between grades 2 and 5, as students move from additive reasoning to *multiplicative reasoning*. Multiplicative reasoning involves: composite groups (i.e., six bags that contain two dozen apples each); transforming units (i.e., price per gallon, miles per hour); concepts of rate, ratio, scale; and Cartesian product (despite the fancy name, this simply is the number of pairs made from sets of items: for example, 7 shirts and 9 pants result in 63 possible outfits).

To build multiplicative reasoning, students explore equal groups, arrays, and area problems. These problems, which can be solved with additive strategies, lay the foundation for the cognitive shift to multiplicative reasoning. Hence, the thinking strategies summarized below begin with what students already know – addition, number relationships, and number composition – and expand to multiplicative thinking as their reasoning develops.

Addition:

Use addition to solve multiplication.

Skip Count
Example: 7 x 5 = ?
Solution: Count by five's 7 times: 5, 10, 15, 20, 25, 30, 35

Repetitive Addition
Example: 3 x 8 = ?
Solution: 8 + 8 + 8 = 24

Anchor Facts

Use well known multiplication facts to solve unknowns. Particularly useful for solving the more difficult x7, x 9, and x12 problems are using the easier x2, x5 and x10 facts.

7s Strategy: Times 7 = Times 5 + Times 2

Example: 8 x 7 = I know (8 x 5) and (8 x 2)
= 40 + 16
= 56

9s Strategy: Times 9 = Times 10 – Times 1

Example: 9 x 6 = I know (10 x 6) is 60.
= 60 – 6
= 54

12s Strategy: Times 12 = Times 10 + Times 2

Example: 11 x 12 = I do know (11 x 10) and (11 x 2)
= 110 + 22
= 132

Unknown Facts: Build From Facts You Do Know

Examples: 6 x 7 = ? Hmm, I know 6 x 6 = 36, so another 6 makes 42.
4 x 9 = ? I know 2 x 9 is 18, so double this and get 36.
13 x 12 = ? Let's see, 13 x 10 = 130 and 13 x 2 = 26, so 130 + 26= 156.
6 x 19 = ? Well, 6 x 20 = 120 and 120-6 = 114.

Basic Facts

Division Strategies

As students explore number relationships and patterns in division, they make generalizations about how division works and apply this knowledge to their computation.

Key Terms	**Dividend** ↓ **Divisor** ↓
	16 ÷ 8 = 2
To divide means to share items equally; to separate quantities into "fair shares."	↑ **Quotient**

I. Basic properties of division:

- **0 ÷ N = 0**: **Zero divided by a number is 0**.
- **N ÷ N = 1**: **A number divided by itself is 1**.
- **N ÷ 1 = N**: **A number divided by one equals the number.**

N ÷ 0 = can not be done. [If N ÷ 0 = ☐ was possible, then it would follow that ☐ x 0 = N but this is impossible.]

II. Division strategies:

- **Use Double Facts** — **Solve N ÷ 2 problems with double facts.**

  12 ÷ 2 = 6 because two 6s make 12.

  18 ÷ 2 = 9 because two 9s make 18.

- **Think Multiplication:** — **Solve division problems with multiplication.**

  24 ÷ 6 = ?
  Think *6 x what = 24?* It's 4.
  24 ÷ 6 = 4

  54 ÷ 9 = ?
  Think *9 x what = 54?* That's 6.
  54 ÷ 9 = 6

- **Key Terms** -

Commutative Property: Factors can be multiplied in any order without changing the product.

Factor: The numbers being multiplied.

Product: The result of multiplication.

Remainder: The number left over when a set of items is shared equally.

Quotient: The result of division.